This Essay revolves around the investigation of counting the numbers of solutions to . As a generalization to the cases of prime numbers, this essay delivers a generalized way of counting number solutions that are smaller than n for composite numbers.

Understanding modular arithmetic is fundamental in number theory, and one of the interesting properties to explore is the set of numbers less than n that satisfy the congruence equation . It is well-established that when n is prime or a prime power, the only solutions are However, when n is composite, the situation becomes more intricate. To illustrate an easy example could be when n=15 a=11.

A natural question arises: are the only solutions, or do additional values satisfy the equation? For instance, when, we find that when n=15 a=11 the equation is also satisfied.

By utilizing computational simulations, we gathered data for various values of n. For example, when n=15 the count of valid solutions is 4; for n=45 it is also 4; even when n=32805 it is still 4; and when we adjust the value of n to be 30, it becomes 8. This pattern suggests that the count consistently follows a power of 2, specifically , where k represents the number of distinct prime factors of n. This observation aligns with our understanding of the cases when n is prime or a prime power, motivating a deeper investigation into its proof and generalization.

The initiation for this investigation originated from an attempt to prove Gauss' extension of Wilson's theorem. The proof involves identifying numbers within the range 1 to n that had the number itself serving as its own modular inverse. According to Gauss' extension, the count of such numbers should be a multiple of 4 for certain forms of composite numbers.

Through both computational evidence and rigorous proof, we have established a generalized formula for counting the natural numbers less than n that satisfy . This result could be expanded for more equations in the form of

Which could reveal more applications in a variety of fields and problems.